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KC-6001

B. E. - I (Sem. I) (All) Examination

November / December – 2012

Engineering Mathematics - I

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. E. - 1 (SEM. 1) (ALL)**

Name of the Subject :

☛ **ENGINEERING MATHEMATICS - 1**

☛ Subject Code No. : **6 0 0 1** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures on the right indicate marks.

(4) Draw the figure whenever it is necessary.

1 (a) Do as directed : **10**

(1) State both Taylor's and Maclaurin's series of one variable.

(2) If $\log(2x+3)$ then find y_n.

(3) Evaluate $\lim_{a \to b} \frac{a^x - b^x}{a^y - b^y}$.

(4) Define relation between circular and hyperbolic functions.

(5) Evaluate $\log(-i)$.

(b) State and prove Leibnitz's theorem for the n^{th} derivative of the product of two functions. **4**

(c) Attempt the following : **6**

(1) Expand $\log \sin x$ in powers of $(x-2)$.

(2) Prove that

$$\tan^{-1}(x+h) = \tan^{-1} x + (h \sin\alpha)\sin\alpha - (h\sin\alpha)^2 \frac{\sin 2\alpha}{2} + \ldots\ldots\ldots\ldots + (-1)^{n-1}$$

where, $\alpha = \cot^{-1} x$.

2 (a) Find the radius of curvature at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$ on the curve $\sqrt{x} + \sqrt{y} = 1$. **5**

(b) Obtain the Maclaurin's series for the function $\sin x$ and $\cos 2x$. **4**

(c) Attempt any two of the following : **6**

(1) $\lim\limits_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$

(2) $\lim\limits_{x\to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$

(3) $\lim\limits_{x\to a} (a-x)\tan\left(\frac{\pi x}{2a}\right)$

3 (a) State and prove De Moivre's theorem. **4**

(b) Attempt any two of the following : **8**

(1) If n is a positive integer then prove that

$$\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = 2^{n+1}\cos\left(\frac{n\pi}{6}\right).$$

(2) $\left(1-e^{i\theta}\right)^{-\frac{1}{2}} + \left(1-e^{-i\theta}\right)^{-\frac{1}{2}} = \sqrt{1+\cos ec\left(\frac{\theta}{2}\right)}$

(3) Prove that $\left\{\sin(\alpha+\beta) - e^{i\alpha}\sin\beta\right\}^n = \sin^n\alpha . e^{-n\beta i}$.

(c) Prove that $\sin^{-1} z = \log\left(z + \sqrt{z^2-1}\right)$. **3**

4 (a) Do as directed : **10**

(1) Define Exact differential equation.

(2) Define radius of curvature.

(3) Define first order and first degree of differential equation.

(4) Define asymptote.

(5) Give the area of the curve in the Cartesian system.

(b) Trace the curve $x^3 + y^3 = 3axy$. **5**

(c) Attempt the following : **4**

(1) Find the perimeter of the closed loop of the curve $9ay^2 = x(x-3a)^2$.

(2) Find the area bounded by X-axis and one arc of the curve $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$.

5 (a) Solve any three of the following : **9**

(1) $\frac{dy}{dx} = e^{x+y} + e^y x^3$

(2) $xdy - ydx = \sqrt{x^2 + y^2}\,dx$

(3) $\frac{dy}{dx} + \frac{1}{x}y = x^2 y^6$

(4) $\left(\sec x \tan x \tan y - e^x\right)dx + \sec x \sec^2 y dy = 0$.

(b) Attempt any two of the following : **6**

(1) $y = 2px + y^2 p^3$

(2) $y - 2px = f\left(xp^2\right)$

(3) $\sin px \cos y = \cos px \sin y + p$

6 Attempt any two of the following : **16**

(1) State and formulate SIR-model. Obtain its solution and analyze it.

(2) Solve $L\frac{di}{dt}+Ri=200\cos 300t$, when $R=100$, $L=0.05$ and find i given that $i=0$ when $t=0$. What value does i approach after a long time ?

(3) In a simple circuit the resistance 2Ω, inductance 0.2 Henry and the e.m.f. be $2\sin 500t$ volts. Find the transient or damping part of the solution.
